SL Paper 1

Consider the simultaneous linear equations

$$x + z = -1$$
$$3x + y + 2z = 1$$
$$2x + ay - z = b$$

where a and b are constants.

a. Using row reduction, find the solutions in terms of a and b when $a \neq 3$. [8]

b. Explain why the equations have no unique solution when $a = 3$. [1]

c. Find all the solutions to the equations when $a = 3$, $b = 10$ in the form $\boldsymbol{r} = \boldsymbol{s} + \lambda \boldsymbol{t}$. [4]

Markscheme

a. $\begin{pmatrix} 1 & 0 & 1 & -1 \\ 3 & 1 & 2 & 1 \\ 2 & a & -1 & b \end{pmatrix} \Rightarrow \begin{pmatrix} 1 & 0 & 1 & -1 \\ 3a-2 & 0 & 2a+1 & a-b \\ 2 & a & -1 & b \end{pmatrix}$ or equivalent ***M1A1***

$$\begin{pmatrix} 1 & 0 & a-3 & -4a+2+b \\ 3a-2 & 0 & 2a+1 & a-b \\ 2 & a & -1 & b \end{pmatrix}$$

$z = \frac{-4a+b+2}{a-3}$ ***M1A1***

$x = -1 - z$ ***M1***

$x = -1 - \left(\frac{-4a+b+2}{a-3}\right)$

$x = \frac{-a+3+4a-b-2}{a-3}$

$x = \frac{3a-b+1}{a-3}$ ***A1***

$y = 1 - 3x - 2z$ ***M1***

$y = 1 - 3\left(\frac{3a-b+1}{a-3}\right) - 2\left(\frac{-4a+b+2}{a-3}\right)$

$= \frac{a-3-9a+3b-3+8a-2b-4}{a-3}$

$= \frac{b-10}{a-3}$ ***A1***

[8 marks]

b. when $a = 3$ the denominator of x, y and $z = 0$ ***R1***

Note: Accept any valid reason.

hence no unique solutions ***AG***

[1 mark]

c. For example let $z = \lambda$ ***(M1)***

$x = -1 - \lambda$ ***(A1)***

$y = 1 - 3(-1 - \lambda) - 2\lambda$

$y = 4 + \lambda$ ***(A1)***

$r = \begin{pmatrix} -1 \\ 4 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}$ ***A1***

[4 marks]

Note: Accept answers which let $x = \lambda$ or $y = \lambda$.

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]

Consider the matrix ***M*** = $\begin{bmatrix} 2 & -4 \\ -1 & -1 \end{bmatrix}$.

a. Show that the linear transformation represented by ***M*** transforms any point on the line $y = x$ to a point on the same line. [2]

b. Explain what happens to points on the line $4y + x = 0$ when they are transformed by ***M***. [3]

c. State the two eigenvalues of ***M***. [2]

d. State two eigenvectors of ***M*** which correspond to the two eigenvalues. [2]

Markscheme

a. $\begin{pmatrix} 2 & -4 \\ -1 & -1 \end{pmatrix} \begin{pmatrix} k \\ k \end{pmatrix} = \begin{pmatrix} -2k \\ -2k \end{pmatrix} \left(= -2 \begin{pmatrix} k \\ k \end{pmatrix} \right)$ ***M1A1***

hence still on the line $y = x$ ***AG***

[2 marks]

b. consider $\begin{pmatrix} 2 & -4 \\ -1 & -1 \end{pmatrix} \begin{pmatrix} 4k \\ -k \end{pmatrix}$ ***M1***

$= \begin{pmatrix} 12k \\ -3k \end{pmatrix} \left(= 3 \begin{pmatrix} 4k \\ -k \end{pmatrix} \right)$ ***A1***

hence the line is invariant ***A1***

[3 marks]

c. hence the eigenvalues are –2 and 3 ***A1A1***

[2 marks]

d. $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 4 \\ -1 \end{pmatrix}$ or equivalent ***A1A1***

[2 marks]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]
d. [N/A]

A matrix ***M*** is called idempotent if $\boldsymbol{M}^2 = \boldsymbol{M}$.

The idempotent matrix ***N*** has the form

$$\boldsymbol{N} = \begin{pmatrix} a & -2a \\ a & -2a \end{pmatrix}$$

where $a \neq 0$.

a. (i) Explain why ***M*** is a square matrix. [4]

(ii) Find the set of possible values of det(***M***).

b. (i) Find the value of a. [12]

(ii) Find the eigenvalues of ***N***.

(iii) Find corresponding eigenvectors.

Markscheme

a. (i) $\boldsymbol{M}^2 = \boldsymbol{MM}$ only exists if the number of columns of ***M*** equals the number of rows of ***M*** ***R1***

hence ***M*** is square ***AG***

(ii) apply the determinant function to both sides ***M1***

$\det(\boldsymbol{M}^2) = \det(\boldsymbol{M})$

use the multiplicative property of the determinant

$\det(\boldsymbol{M}^2) = \det(\boldsymbol{M})\ \det(\boldsymbol{M}) = \det(\boldsymbol{M})$ ***(M1)***

hence $\det(\boldsymbol{M}) = 0$ or 1 ***A1***

[4 marks]

b. (i) attempt to calculate $\boldsymbol{N}^2$ ***M1***

obtain $\begin{pmatrix} -a^2 & 2a^2 \\ -a^2 & 2a^2 \end{pmatrix}$ ***A1***

equating to ***N*** ***M1***

to obtain $a = -1$ ***A1***

(ii) $\boldsymbol{N} = \begin{pmatrix} -1 & 2 \\ -1 & 2 \end{pmatrix}$

$\boldsymbol{N} - \lambda \boldsymbol{I} = \begin{pmatrix} -1-\lambda & 2 \\ -1 & 2-\lambda \end{pmatrix}$ ***M1***

$(-1-\lambda)(2-\lambda)+2=0$ ***(A1)***

$\lambda^2-\lambda=0$ ***(A1)***

λ is 1 or 0 ***A1***

(iii) let $\lambda=1$

to obtain $\begin{pmatrix}-1 & 2\\ -1 & 2\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}=\begin{pmatrix}x\\ y\end{pmatrix}$ or $\begin{pmatrix}-2 & 2\\ -1 & 1\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}=\begin{pmatrix}0\\ 0\end{pmatrix}$ ***M1***

hence eigenvector is $\begin{pmatrix}x\\ x\end{pmatrix}$ ***A1***

let $\lambda=0$

to obtain $\begin{pmatrix}-1 & 2\\ -1 & 2\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix}=\begin{pmatrix}0\\ 0\end{pmatrix}$ ***M1***

hence eigenvector is $\begin{pmatrix}2y\\ y\end{pmatrix}$ ***A1***

Note: Accept specific eigenvectors.

[12 marks]

Examiners report

a. This was a more successful question for many candidates with a number of fully correct solutions being seen and a significant number of partially correct answers. Most candidates understood what was required from part (a)(i), but part (ii) often resulted in unnecessarily complex algebra which they were unable to manipulate. Part (b) resulted in many wholly successful answers, provided candidates realised the need for care in terms of the manipulation.

b. This was a more successful question for many candidates with a number of fully correct solutions being seen and a significant number of partially correct answers. Most candidates understood what was required from part (a)(i), but part (ii) often resulted in unnecessarily complex algebra which they were unable to manipulate. Part (b) resulted in many wholly successful answers, provided candidates realised the need for care in terms of the manipulation.

Let $\boldsymbol{A}^2 = 2\boldsymbol{A} + \boldsymbol{I}$ where $\boldsymbol{A}$ is a 2 × 2 matrix.

a. Show that $\boldsymbol{A}^4 = 12\boldsymbol{A} + 5\boldsymbol{I}$. [3]

b. Let $\boldsymbol{B} = \begin{bmatrix}4 & 2\\ 1 & -3\end{bmatrix}$. [3]

Given that $\boldsymbol{B}^2 - \boldsymbol{B} - 4\boldsymbol{I} = \begin{bmatrix}k & 0\\ 0 & k\end{bmatrix}$, find the value of k.

Markscheme

a. **METHOD 1**

$\boldsymbol{A}^4 = 4\boldsymbol{A}^2 + 4\mathrm{A}\boldsymbol{I} + \boldsymbol{I}^2$ or equivalent ***M1A1***

$= 4(2\boldsymbol{A} + \boldsymbol{I}) + 4\boldsymbol{A} + \boldsymbol{I}$ ***A1***

$= 8\boldsymbol{A} + 4\mathrm{I} + 4\boldsymbol{A} + \boldsymbol{I}$

$= 12\boldsymbol{A} + 5\boldsymbol{I}$ ***AG***

[3 marks]

METHOD 2

$\boldsymbol{A}^3 = \boldsymbol{A}(2\boldsymbol{A} + \boldsymbol{I}) = 2\boldsymbol{A}^2 + \boldsymbol{AI} = 2(2\boldsymbol{A} + \boldsymbol{I}) + \boldsymbol{A}(= 5\boldsymbol{A} + 2\boldsymbol{I})$ ***M1A1***

$\boldsymbol{A}^4 = \boldsymbol{A}(5\boldsymbol{A} + 2\boldsymbol{I})$ ***A1***

$= 5\boldsymbol{A}^2 + 2\boldsymbol{A} = 5(2\boldsymbol{A} + \boldsymbol{I}) + 2\boldsymbol{A}$

$= 12\boldsymbol{A} + 5\boldsymbol{I}$ ***AG***

[3 marks]

b. $\boldsymbol{B}^2 = \begin{bmatrix} 18 & 2 \\ 1 & 11 \end{bmatrix}$ ***(A1)***

$\begin{bmatrix} 18 & 2 \\ 1 & 11 \end{bmatrix} - \begin{bmatrix} 4 & 2 \\ 1 & -3 \end{bmatrix} - \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix} = \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}$ ***(A1)***

$\Rightarrow k = 10$ ***A1***

[3 marks]

Examiners report

a. [N/A]
b. [N/A]

Consider the system of equations

$$\begin{bmatrix} 1 & 2 & 1 & 3 \\ 2 & 1 & 3 & 1 \\ 5 & 1 & 8 & 0 \\ 3 & 3 & 4 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ \lambda \\ \mu \end{bmatrix}$$

a. Determine the value of λ and the value of μ for which the equations are consistent. [5]

b. For these values of λ and μ, solve the equations. [3]

c. State the rank of the matrix of coefficients, justifying your answer. [2]

Markscheme

a. using row operations on 4×5 matrix, ***M1***

$$\begin{bmatrix}1 & 2 & 1 & 3\\ 0 & -3 & 1 & -5\\ 0 & -9 & 3 & -15\\ 0 & -3 & 1 & -5\end{bmatrix}\begin{bmatrix}2\\ -1\\ \lambda-10\\ \mu-6\end{bmatrix}\begin{matrix}\\ \text{row2} - 2\times\text{row1}\\ \text{row3} - 5\times\text{row1}\\ \text{row4} - 3\times\text{row1}\end{matrix}$$ ***A2***

or any alternative correct row reductions

Note: Award ***A1*** for two correct row reductions.

$\lambda = 7$ ***A1***

$\mu = 5$ ***A1***

[5 marks]

b. let $x_3 = \alpha,\ x_4 = \beta$ ***M1***

$x_2 = \frac{1+\alpha-5\beta}{3}$ ***A1***

$x_1 = \frac{4-5\alpha+\beta}{3}$ ***A1***

Note: Alternative solutions are available.

[3 marks]

c. the rank is 2 ***A1***

because the matrix has 2 independent rows or a correct comment based on the use of rref ***R1***

[2 marks]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]

The non-zero vectors $\boldsymbol{v}_1$, $\boldsymbol{v}_2$, $\boldsymbol{v}_3$ form an orthogonal set of vectors in $\mathbb{R}^3$.

a.i. By considering $\alpha_1\boldsymbol{v}_1 + \alpha_2\boldsymbol{v}_2 + \alpha_3\boldsymbol{v}_3 = 0$, show that $\boldsymbol{v}_1$, $\boldsymbol{v}_2$, $\boldsymbol{v}_3$ are linearly independent. [3]

a.ii. Explain briefly why $\boldsymbol{v}_1$, $\boldsymbol{v}_2$, $\boldsymbol{v}_3$ form a basis for vectors in $\mathbb{R}^3$. [3]

b.i. Show that the vectors [2]

$$\begin{bmatrix}1\\0\\1\end{bmatrix};\ \begin{bmatrix}-1\\1\\1\end{bmatrix};\ \begin{bmatrix}1\\2\\-1\end{bmatrix}$$

form an orthogonal basis.

b.ii. Express the vector [3]

$$\begin{bmatrix} 2 \\ 8 \\ 0 \end{bmatrix}$$

as a linear combination of these vectors.

Markscheme

a.i. let $\alpha_1\boldsymbol{v}_1 + \alpha_2\boldsymbol{v}_2 + \alpha_3\boldsymbol{v}_3 = 0$

take the dot product with $\boldsymbol{v}_1$ ***M1***

$\alpha_1\boldsymbol{v}_1 \bullet \boldsymbol{v}_1 + \alpha_2\boldsymbol{v}_2 \bullet \boldsymbol{v}_1 + \alpha_3\boldsymbol{v}_3 \bullet \boldsymbol{v}_1 = 0$ ***A1***

because the vectors are orthogonal, $\boldsymbol{v}_2 \bullet \boldsymbol{v}_1 = \boldsymbol{v}_3 \bullet \boldsymbol{v}_1 = 0$ ***R1***

and since $\boldsymbol{v}_1 \bullet \boldsymbol{v}_1 > 0$ it follows that $\alpha_1 = 0$ and similarly, $\alpha_2 = \alpha_3 = 0$ ***R1***

so $\alpha_1\boldsymbol{v}_1 + \alpha_2\boldsymbol{v}_2 + \alpha_3\boldsymbol{v}_3 = 0 \Rightarrow \alpha_1 = \alpha_2 = \alpha_3 = 0$ therefore $\boldsymbol{v}_1, \boldsymbol{v}_2, \boldsymbol{v}_3$, are linearly independent ***R1AG***

[3 marks]

a.ii. the three vectors form a basis for $\mathbb{R}^3$ because they are (linearly) independent ***R1***

[3 marks]

b.i. $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \bullet \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix} = 0$; $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \bullet \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix} = 0$; $\begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix} \bullet \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix} = 0$ ***M1A1***

therefore the vectors form an orthogonal basis ***AG***

[??? marks]

b.ii. let $\begin{bmatrix} 2 \\ 8 \\ 0 \end{bmatrix} = \lambda \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + \mu \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix} + v \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$ ***M1***

$\lambda - \mu + v = 2$

$\mu + 2v = 8$

$\lambda = \mu - v = 0$ ***A1***

the solution is

$\begin{bmatrix} \lambda \\ \mu \\ v \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \quad \left(\begin{bmatrix} 2 \\ 8 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + 2\begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix} + 3\begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}\right)$ ***A1***

[??? marks]

Examiners report

a.i. [N/A]
a.ii. [N/A]
b.i. [N/A]
b.ii. [N/A]

In this question, x, y and z denote the coordinates of a point in three-dimensional Euclidean space with respect to fixed rectangular axes with origin O. The vector space of position vectors relative to O is denoted by $\mathbb{R}^3$.

a. Explain why the set of position vectors of points whose coordinates satisfy $x - y - z = 1$ does not form a vector subspace of $\mathbb{R}^3$. [1]

b. (i) Show that the set of position vectors of points whose coordinates satisfy $x - y - z = 0$ forms a vector subspace, V, of $\mathbb{R}^3$. [13]

(ii) Determine an orthogonal basis for V of which one member is $\begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$.

(iii) Augment this basis with an orthogonal vector to form a basis for $\mathbb{R}^3$.

(iv) Express the position vector of the point with coordinates $(4,\ 0,\ -2)$ as a linear combination of these basis vectors.

Markscheme

a. Accept any valid reasoning:

Example 1:

$(1,\ 0,\ 0)$ lies on the plane, however linear combinations of this do not (for example $(2,\ 0,\ 0)$) ***R1***

hence the position vectors of the points on the plane do not form a vector space ***AG***

Example 2:

the given plane does not pass through the origin (or the zero vector is not the position vector of any point on the plane) ***R1***

hence the position vectors of the points on the plane do not form a vector space ***AG***

[1 mark]

b. (i) (the set of position vectors is non-empty)

let $x_1 = \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix}$ be the position vector of a point on the plane and $a \in \mathbb{R}$

then the coordinates of the position vector of ax satisfy the equation for the plane because $ax_1 - ay_1 - az_1 = a(x_1 - y_1 - z_1) = 0$ ***M1A1***

let $x_2 = \begin{pmatrix} x_2 \\ y_2 \\ x_2 \end{pmatrix}$ be the position vector of another point on the plane

consider $x_3 = x_1 + x_2$

then the coordinates of $x_3 = \begin{pmatrix} x_3 = x_1 + x_2 \\ y_1 = y_1 + y_2 \\ z_3 = z_1 + z_2 \end{pmatrix}$ satisfy ***M1***

$x_3 - y_3 - z_3 = (x_1 + x_2) - (y_1 + y_2) - (z_1 + z_2)$

$= 0$ ***A1***

subspace conditions established ***AG***

Note: The above conditions may be combined in one calculation.

(ii) if $\begin{pmatrix} a \\ b \\ c \end{pmatrix}$ is the position vector of a second point on the plane orthogonal to the given vector, then ***(M1)***

$a - b - c = 0$ and $a + 2b - c = 0$ ***(A1)(A1)***

for example $\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$ completes the basis ***A1***

(iii) the basis for $(\mathbb{R}^3)$ can be augmented to an orthogonal basis for $\mathbb{R}^3$ by adjoining $\begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} \times \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$ ***(M1)***

$= \begin{pmatrix} 2 \\ -2 \\ -2 \end{pmatrix}$ ***A1***

(iv) attempt to solve $\alpha = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} + \beta \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + \gamma \begin{pmatrix} 2 \\ -2 \\ -2 \end{pmatrix} = \begin{pmatrix} 4 \\ 0 \\ -2 \end{pmatrix}$ ***M1***

obtain $\alpha = \beta = \gamma = 1$ ***A2***

[13 marks]

Examiners report

a. Again this was found difficult by many candidates and resulted in no attempt being made. For those who were able to start, parts (a) and (b)(i) showed a reasonable degree of understanding. After that it was only a significant minority of candidates who were able to proceed successfully with many ignoring or not realising the significance of the word "orthogonal".

b. Again this was found difficult by many candidates and resulted in no attempt being made. For those who were able to start, parts (a) and (b)(i) showed a reasonable degree of understanding. After that it was only a significant minority of candidates who were able to proceed successfully with many ignoring or not realising the significance of the word "orthogonal".

The matrix $\boldsymbol{A}$ is given by $\boldsymbol{A} = \begin{pmatrix} 1 & 2 & 1 \\ 1 & 1 & 2 \\ 2 & 3 & 1 \end{pmatrix}$.

(a) Given that $\boldsymbol{A}^3$ can be expressed in the form $\boldsymbol{A}^3 = a\boldsymbol{A}^2 = b\boldsymbol{A} + c\boldsymbol{I}$, determine the values of the constants a, b, c.

(b) (i) Hence express $\boldsymbol{A}^{-1}$ in the form $\boldsymbol{A}^{-1} = d\boldsymbol{A}^2 = e\boldsymbol{A} + f\boldsymbol{I}$ where $d,\ e,\ f \in \mathbb{Q}$.

(ii) Use this result to determine $\boldsymbol{A}^{-1}$.

Markscheme

(a) successive powers of $\boldsymbol{A}$ are given by

$\boldsymbol{A}^2 = \begin{pmatrix} 5 & 7 & 6 \\ 6 & 9 & 5 \\ 7 & 10 & 9 \end{pmatrix}$ ***(M1)A1***

$\boldsymbol{A}^3 = \begin{pmatrix} 24 & 35 & 25 \\ 25 & 36 & 29 \\ 35 & 51 & 36 \end{pmatrix}$ ***A1***

it follows, considering elements in the first rows, that

$5a + b + c = 24$

$7a + 2b = 35$

$6a + b = 25$ ***M1A1***

solving, ***(M1)***

$(a,\ b,\ c) = (3,\ 7,\ 2)$ ***A1***

Note: Accept any other three correct equations.

Note: Accept the use of the Cayley–Hamilton Theorem.

[7 marks]

(b) (i) it has been shown that

$\boldsymbol{A}^3 = 3\boldsymbol{A}^2 + 7\boldsymbol{A} + 2\boldsymbol{I}$

multiplying by $\boldsymbol{A}^{-1}$, ***M1***

$\boldsymbol{A}^2 = 3\boldsymbol{A} + 7\boldsymbol{I} + 2\boldsymbol{A}^{-1}$ ***A1***

whence

$\boldsymbol{A}^{-1} = 0.5\boldsymbol{A}^2 - 1.5\boldsymbol{A} - 3.5\boldsymbol{I}$ ***A1***

(ii) substituting powers of $\boldsymbol{A}$,

$\boldsymbol{A}^{-1} = 0.5\begin{pmatrix} 5 & 7 & 6 \\ 6 & 9 & 5 \\ 7 & 10 & 9 \end{pmatrix} - 1.5\begin{pmatrix} 1 & 2 & 1 \\ 1 & 1 & 2 \\ 2 & 3 & 1 \end{pmatrix} - 3.5\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ ***M1***

$= \begin{pmatrix} -2.5 & 0.5 & 1.5 \\ 1.5 & -0.5 & -0.5 \\ 0.5 & 0.5 & -0.5 \end{pmatrix}$ ***A1***

Note: Follow through their equation in (b)(i).

Note: Line (ii) of (ii) must be seen.

[5 marks]

Examiners report

[N/A]

A transformation T is a linear mapping from $\mathbb{R}^3$ to $\mathbb{R}^4$, represented by the matrix

$$M = \begin{pmatrix} 1 & 2 & 1 \\ 2 & 7 & 5 \\ -3 & 1 & 4 \\ 1 & 5 & 4 \end{pmatrix}$$

a. (i) Find the row rank of M. [8]

(ii) Hence or otherwise find the kernel of T.

b. (i) State the column rank of M. [4]

(ii) Find the basis for the range of this transformation.

Markscheme

a. (i) row reduction gives $\begin{pmatrix} 1 & 2 & 1 \\ 0 & 3 & 3 \\ 0 & 7 & 7 \\ 0 & 3 & 3 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 2 & 1 \\ 0 & 3 & 3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \left(\rightarrow \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \right)$ ***(M1)A1***

hence row rank is 2 ***A1***

Note: Accept the argument that Column 2 = Column 1 + Column 3

(ii) to find the kernel $\begin{pmatrix} 1 & 2 & 1 \\ 0 & 3 & 3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$ ***M1***

Note: Allow the use of the original matrix

$x + 2y + z = 0$

$3y + 3z = 0$ ***A1***

let $z = \lambda$ ***M1***

hence $y = -\lambda,\ x = \lambda$

the kernel is therefore $\begin{bmatrix} \lambda \\ -\lambda \\ \lambda \end{bmatrix}$ ***A1***

b. (i) column rank is 2 ***A1***

(ii) a basis for the range is defined by two independent vectors ***(M1)***

therefore a basis for the range is for example, $\begin{bmatrix} 1 \\ 2 \\ -3 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ 7 \\ 1 \\ 5 \end{bmatrix}$ ***A2***

Examiners report

a. Many solutions to this question suggested that the topic had not been adequately covered in many centres so that solutions were either good or virtually non existent. Most successful candidates used their calculator to perform the row reduction.

b. Many solutions to this question suggested that the topic had not been adequately covered in many centres so that solutions were either good or virtually non existent. Most successful candidates used their calculator to perform the row reduction.

Let $\boldsymbol{S}$ be the set of matrices given by

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix};\ a, b, c, d \in \mathbb{R}, ad - bc = 1$$

The relation R is defined on S as follows. Given $\boldsymbol{A}$, $\boldsymbol{B} \in S$, $\boldsymbol{ARB}$ if and only if there exists $\boldsymbol{X} \in S$ such that $\boldsymbol{A} = \boldsymbol{BX}$.

a. Show that R is an equivalence relation. [8]

b. The relationship between a , b , c and d is changed to $ad - bc = n$. State, with a reason, whether or not there are any non-zero values of n , other than 1, for which R is an equivalence relation. [2]

Markscheme

a. since $\boldsymbol{A} = \boldsymbol{AI}$ where $\boldsymbol{I}$ is the identity *A1*

and $\det(\boldsymbol{I}) = 1$, *A1*

R is reflexive

$\boldsymbol{ARB} \Rightarrow \boldsymbol{A} = \boldsymbol{BX}$ where $\det(\boldsymbol{X}) = 1$ *M1*

it follows that $\boldsymbol{B} = \boldsymbol{AX}^{-1}$ *A1*

and $\det(\boldsymbol{X}^{-1}) = \det(\boldsymbol{X})^{-1} = 1$ *A1*

R is symmetric

$\boldsymbol{ARB}$ and $\boldsymbol{BRC} \Rightarrow \boldsymbol{A} = \boldsymbol{BX}$ and $\boldsymbol{B} = \boldsymbol{CY}$ where $\det(\boldsymbol{X}) = \det(\boldsymbol{Y}) = 1$ *M1*

it follows that $\boldsymbol{A} = \boldsymbol{CYX}$ *A1*

$\det(\boldsymbol{YX}) = \det(\boldsymbol{Y})\det(\boldsymbol{X}) = 1$ *A1*

R is transitive

hence R is an equivalence relation *AG*

[8 marks]

b. for reflexivity, we require $\boldsymbol{ARA}$ so that $\boldsymbol{A} = \boldsymbol{AI}$ (for all $\boldsymbol{A} \in S$) *M1*

since $\det(\boldsymbol{I}) = 1$ and we require $\boldsymbol{I} \in S$ the only possibility is $n = 1$ *A1*

[2 marks]

Examiners report

a. This question was not well done in general, again illustrating that questions involving both matrices and equivalence relations tend to cause problems for candidates. A common error was to assume, incorrectly, that ARB and BRC $\Rightarrow A = BX$ and $B = CX$, not realizing that a different "x" is required each time. In proving that R is an equivalence relation, consideration of the determinant is necessary in this question although many candidates neglected to do this.

b. In proving that R is an equivalence relation, consideration of the determinant is necessary in this question although many candidates neglected to do this.

The matrix $\boldsymbol{M}$ is defined by $\boldsymbol{M} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$.

The eigenvalues of $\boldsymbol{M}$ are denoted by $\lambda_1,\ \lambda_2$.

(a) Show that $\lambda_1 + \lambda_2 = a + d$ and $\lambda_1\lambda_2 = \det(\boldsymbol{M})$.

(b) Given that $a + b = c + d = 1$, show that 1 is an eigenvalue of $\boldsymbol{M}$.

(c) Find eigenvectors for the matrix $\begin{pmatrix} 2 & -1 \\ 3 & -2 \end{pmatrix}$.

Markscheme

(a) the eigenvalues satisfy $\begin{vmatrix} a-\lambda & b \\ c & d-\lambda \end{vmatrix} = 0$ ***(M1)***

$\lambda^2 - (a+d)\lambda + ad - bc = 0$ ***A1***

using the sum and product properties of the roots of a quadratic equation ***R1***

$\lambda_1 + \lambda_2 = a + d,\ \lambda_1\lambda_2 = ad - bc = \det(\boldsymbol{M})$ ***AG***

[3 marks]

(b) let $f(\lambda) = \lambda^2 - (a+d)\lambda + ad - bc$

putting $b = 1 - a$ and $d = 1 - c$, consider ***M1***

$f(1) = 1 - a - 1 + c + a - ac - c + ac = 0$ ***A1***

therefore $\lambda = 1$ is an eigenvalue ***AG***

[2 marks]

Note: Allow substitution for b, c into the quadratic equation for λ followed by solution of this equation.

(c) using any valid method ***(M1)***

the eigenvalues are 1 and –1 ***A1***

an eigenvector corresponding to $\lambda = 1$ satisfies

$\begin{pmatrix} 2 & -1 \\ 3 & -2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$ **or** $\begin{pmatrix} 1 & -1 \\ 3 & -3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ ***M1A1***

$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ **or** any multiple ***A1***

an eigenvector corresponding to $\lambda = -1$ satisfies

$\begin{pmatrix} 2 & -1 \\ 3 & -2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = -\begin{pmatrix} x \\ y \end{pmatrix}$ **or** $\begin{pmatrix} 3 & -1 \\ 3 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ ***M1***

$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$ **or** any multiple ***A1***

Note: Award ***M1A1A1*** for calculating the first eigenvector and ***M1A1*** for the second irrespective of the order in which they are calculated.

[7 marks]

Examiners report

[N/A]

The matrix $\boldsymbol{A}$ is given by

$$\boldsymbol{A} = \begin{pmatrix} 0 & 1 & 0 \\ 2 & 4 & 1 \\ 4 & -11 & -2 \end{pmatrix}.$$

a. (i) Find the matrices $\boldsymbol{A}^2$ and $\boldsymbol{A}^3$, and verify that $\boldsymbol{A}^3 = 2\boldsymbol{A}^2 - \boldsymbol{A}$. [6]

(ii) Deduce that $\boldsymbol{A}^4 = 3\boldsymbol{A}^2 - 2\boldsymbol{A}$.

b. (i) Suggest a similar expression for $\boldsymbol{A}^n$ in terms of $\boldsymbol{A}$ and $\boldsymbol{A}^2$, valid for $n \geq 3$. [8]

(ii) Use mathematical induction to prove the validity of your suggestion.

Markscheme

a. (i) $\boldsymbol{A}^2 = \begin{pmatrix} 2 & 4 & 1 \\ 4 & 7 & 2 \\ -14 & -26 & -7 \end{pmatrix}$ ***A1***

$\boldsymbol{A}^3 = \begin{pmatrix} 4 & 7 & 2 \\ 6 & 10 & 3 \\ -24 & -41 & -12 \end{pmatrix}$ ***A1***

$2\boldsymbol{A}^2 - \boldsymbol{A} = 2\begin{pmatrix} 2 & 4 & 1 \\ 4 & 7 & 2 \\ -14 & -26 & -7 \end{pmatrix} - \begin{pmatrix} 0 & 1 & 0 \\ 2 & 4 & 1 \\ -4 & -11 & -2 \end{pmatrix}$ ***M1***

$= \begin{pmatrix} 4 & 7 & 2 \\ 6 & 10 & 3 \\ -24 & -41 & -12 \end{pmatrix} = \boldsymbol{A}^3$ ***AG***

(ii) $\boldsymbol{A}^4 = \boldsymbol{A}\boldsymbol{A}^3$ ***M1***

$= \boldsymbol{A}(2\boldsymbol{A}^2 - \boldsymbol{A})$ ***A1***

$= 2\boldsymbol{A}^3 - \boldsymbol{A}^2$

$= 2(2\boldsymbol{A}^2 - \boldsymbol{A}) - \boldsymbol{A}^2$ ***A1***

$= 3\boldsymbol{A}^2 - 2\boldsymbol{A}$ ***AG***

Note: Accept alternative solutions that include correct calculation of both sides of the expression.

[6 marks]

b. (i) conjecture: $\boldsymbol{A}^n = (n-1)\,\boldsymbol{A}^2 - (n-2)\,\boldsymbol{A}$ ***A1***

(ii) first check that the result is true for $n = 3$

the formula gives $\boldsymbol{A}^3 = 2\boldsymbol{A}^2 - \boldsymbol{A}$ which is correct ***A1***

assume the result for $n = k$, *i.e.* ***M1***

$\boldsymbol{A}^k = (k-1)\mathbf{A}^2 - (k-2)\boldsymbol{A}$

so

$\boldsymbol{A}^{k+1} = \boldsymbol{A}\left[(k-1)\,\boldsymbol{A}^2 - (k-2)\,\boldsymbol{A}\right]$ ***M1***

$= (k-1)\,\boldsymbol{A}^3 - (k-2)\,\boldsymbol{A}^2$ ***A1***

$= (k-1)\left(2\boldsymbol{A}^2 - \boldsymbol{A}\right) - (k-2)\,\boldsymbol{A}^2$ ***M1***

$= k\boldsymbol{A}^2 - (k-1)\,\boldsymbol{A}$ ***A1***

so true for $n = k \Rightarrow$ true for $n = k+1$ and since true for $n = 3$,

the result is proved by induction ***R1***

Note: Only award the ***R1*** mark if a reasonable attempt at a proof by induction has been made.

[8 marks]

Examiners report

a. [N/A]
b. [N/A]

Consider the system of equations

$$\begin{pmatrix} 1 & -1 & 2 \\ 2 & 2 & -1 \\ 3 & 5 & -4 \\ 3 & 1 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 5 \\ 3 \\ 1 \\ k \end{pmatrix}.$$

a. By reducing the augmented matrix to row echelon form, [5]

(i) find the rank of the coefficient matrix;

(ii) find the value of k for which the system has a solution.

b. For this value of k , determine the solution. [3]

Markscheme

a. reducing to row echelon form

$$\left.\begin{array}{ccc|c} 1 & -1 & 2 & 5 \\ 0 & 4 & -5 & -7 \\ 0 & 8 & -10 & -14 \\ 0 & 4 & -5 & k-15 \end{array}\right.$$ ***(M1)(A1)***

$$\left.\begin{array}{ccc|c} 1 & -2 & 2 & 5 \\ 0 & 4 & -5 & -7 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & k-8 \end{array}\right.$$ ***A1***

(i) this shows that the rank of the matrix is 2 ***A1***

(ii) the equations can be solved if $k = 8$ ***A1***

[5 marks]

b. let $z = \lambda$ ***A1***

then $y = \frac{5\lambda-7}{4}$ ***A1***

and $x = \left(5 - 2\lambda + \frac{5\lambda-7}{4} =\right) \frac{13-3\lambda}{4}$ ***A1***

Note: Accept equivalent expressions.

[3 marks]

Examiners report

a. [N/A]
b. [N/A]

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a. Show that the following vectors form a basis for the vector space $\mathbb{R}^3$. [3]

$$\begin{pmatrix}1\\2\\3\end{pmatrix};\begin{pmatrix}2\\3\\1\end{pmatrix};\begin{pmatrix}5\\2\\5\end{pmatrix}$$

b. Express the following vector as a linear combination of the above vectors. [5]

$$\begin{pmatrix}12\\14\\16\end{pmatrix}$$

Markscheme

a.
let $\boldsymbol{A} = \begin{vmatrix}1&2&5\\2&3&2\\3&1&5\end{vmatrix}$ and consider $\det(\boldsymbol{A}) = -30$ ***(M1)A1***

the vectors form a basis because the determinant is non-zero (or because the matrix is non-singular) ***R1***

[3 marks]

b.
let $\begin{pmatrix}12\\14\\16\end{pmatrix} = \lambda\begin{pmatrix}1\\2\\3\end{pmatrix} + \mu\begin{pmatrix}2\\3\\1\end{pmatrix} + v\begin{pmatrix}5\\2\\5\end{pmatrix}$ ***M1A1***

so that

EITHER

$\lambda + 2\mu + 5v = 12$
$2\lambda + 3\mu + 2v = 14$ ***M1***
$3\lambda + \mu + 5v = 16$

OR

$\begin{pmatrix}1&2&5\\2&3&2\\3&1&5\end{pmatrix}\begin{pmatrix}\lambda\\\mu\\v\end{pmatrix} = \begin{pmatrix}12\\14\\16\end{pmatrix}$ ***M1***

THEN

giving $\lambda = 3, \mu = 2, v = 1$ ***(A1)***

hence $\begin{pmatrix}12\\14\\16\end{pmatrix} = 3\begin{pmatrix}1\\2\\3\end{pmatrix} + 2\begin{pmatrix}2\\3\\1\end{pmatrix} + 1\begin{pmatrix}5\\2\\5\end{pmatrix}$ ***A1***

[5 marks]

Examiners report

a. [N/A]
b. [N/A]

The set S contains the eight matrices of the form

$$\begin{pmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{pmatrix}$$

where a, b, c can each take one of the values $+1$ or -1 .

a. Show that any matrix of this form is its own inverse. [3]

b. Show that S forms an Abelian group under matrix multiplication. [9]

c. Giving a reason, state whether or not this group is cyclic. [1]

Markscheme

a. $\begin{pmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{pmatrix}\begin{pmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & c \end{pmatrix} = \begin{pmatrix} a^2 & 0 & 0 \\ 0 & b^2 & 0 \\ 0 & 0 & c^2 \end{pmatrix}$ ***A1M1***

$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ ***A1***

this shows that each matrix is self-inverse

[3 marks]

b. closure:

$\begin{pmatrix} a_1 & 0 & 0 \\ 0 & b_1 & 0 \\ 0 & 0 & c_1 \end{pmatrix}\begin{pmatrix} a_2 & 0 & 0 \\ 0 & b_2 & 0 \\ 0 & 0 & c_2 \end{pmatrix} = \begin{pmatrix} a_1a_2 & 0 & 0 \\ 0 & b_1b_2 & 0 \\ 0 & 0 & c_1c_2 \end{pmatrix}$ ***M1A1***

$= \begin{pmatrix} a_3 & 0 & 0 \\ 0 & b_3 & 0 \\ 0 & 0 & c_3 \end{pmatrix}$

where each of a_3, b_3, c_3 can only be ± 1 ***A1***

this proves closure

identity: the identity matrix is the group identity ***A1***

inverse: as shown above, every element is self-inverse ***A1***

associativity: this follows because matrix multiplication is associative ***A1***

S is therefore a group ***AG***

Abelian:

$\begin{pmatrix} a_2 & 0 & 0 \\ 0 & b_2 & 0 \\ 0 & 0 & c_2 \end{pmatrix}\begin{pmatrix} a_1 & 0 & 0 \\ 0 & b_1 & 0 \\ 0 & 0 & c_1 \end{pmatrix} = \begin{pmatrix} a_2a_1 & 0 & 0 \\ 0 & b_2b_1 & 0 \\ 0 & 0 & c_2c_1 \end{pmatrix}$ ***A1***

$$\begin{pmatrix} a_1 & 0 & 0 \\ 0 & b_1 & 0 \\ 0 & 0 & c_1 \end{pmatrix} \begin{pmatrix} a_2 & 0 & 0 \\ 0 & b_2 & 0 \\ 0 & 0 & c_2 \end{pmatrix} = \begin{pmatrix} a_1a_2 & 0 & 0 \\ 0 & b_1b_2 & 0 \\ 0 & 0 & c_1c_2 \end{pmatrix} \quad \textit{A1}$$

Note: Second line may have been shown whilst proving closure, however a reference to it must be made here.

we see that the same result is obtained either way which proves commutativity so that the group is Abelian ***R1***

[9 marks]

c. since all elements (except the identity) are of order 2, the group is not cyclic (since S contains 8 elements) ***R1***

[1 mark]

Examiners report

a. [N/A]
b. [N/A]
c. [N/A]

By considering the images of the points (1, 0) and (0, 1),

a.i. determine the 2 × 2 matrix ***P*** which represents a reflection in the line $y = (\tan\theta)\,x$. [3]

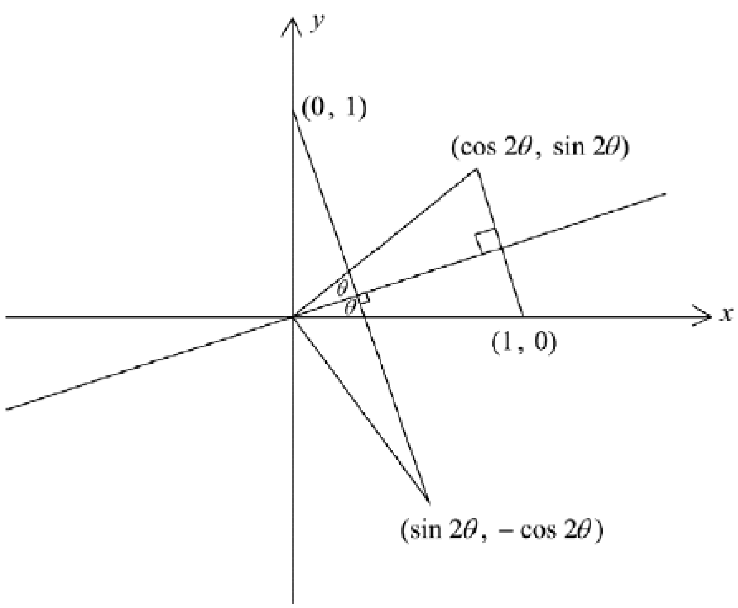
a.ii. determine the 2 × 2 matrix ***Q*** which represents an anticlockwise rotation of θ about the origin. [2]

b. Describe the transformation represented by the matrix ***PQ***. [5]

c. A matrix M is said to be orthogonal if $\boldsymbol{M}^T\boldsymbol{M} = \boldsymbol{I}$ where ***I*** is the identity. Show that ***Q*** is orthogonal. [2]

Markscheme

a.i.



(M1)

using the transformation of the unit square:

$\begin{pmatrix}1\\0\end{pmatrix} \to \begin{pmatrix}\cos 2\theta\\ \sin 2\theta\end{pmatrix}$ and $\begin{pmatrix}0\\1\end{pmatrix} \to \begin{pmatrix}\sin 2\theta\\ -\cos 2\theta\end{pmatrix}$ ***(M1)***

hence the matrix ***P*** is $\begin{pmatrix}\cos 2\theta & \sin 2\theta\\ \sin 2\theta & -\cos 2\theta\end{pmatrix}$ ***A1***

[3 marks]

a.ii.using the transformation of the unit square:

$\begin{pmatrix}1\\0\end{pmatrix} \to \begin{pmatrix}\cos \theta\\ \sin \theta\end{pmatrix}$ and $\begin{pmatrix}0\\1\end{pmatrix} \to \begin{pmatrix}-\sin \theta\\ \cos \theta\end{pmatrix}$ ***(M1)***

hence the matrix ***Q*** is $\begin{pmatrix}\cos \theta & -\sin \theta\\ \sin \theta & \cos \theta\end{pmatrix}$ ***A1***

[2 marks]

b. ***PQ*** = $\begin{pmatrix}\cos\theta\cos 2\theta + \sin\theta\sin 2\theta & \cos\theta\sin 2\theta - \sin\theta\cos 2\theta\\ -\cos 2\theta\sin\theta + \sin 2\theta\cos\theta & -\sin\theta\sin 2\theta - \cos\theta\cos 2\theta\end{pmatrix}$ ***M1A1***

$= \begin{pmatrix}\cos\ (2\theta-\theta) & \sin\ (2\theta-\theta)\\ \sin\ (2\theta-\theta) & -\cos\ (2\theta-\theta)\end{pmatrix} = \begin{pmatrix}\cos\theta & \sin\theta\\ \sin\theta & -\cos\theta\end{pmatrix}$ ***M1A1***

this is a reflection in the line $y = \left(\tan\frac{1}{2}\theta\right)x$ ***A1***

[5 marks]

c. ***Q***^{T}***Q*** = $\begin{pmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{pmatrix}\begin{pmatrix}\cos\theta & -\sin\theta\\ \sin\theta & \cos\theta\end{pmatrix}$

$= \begin{pmatrix}\cos^2\theta + \sin^2\theta & -\cos\theta\sin\theta + \cos\theta\sin\theta\\ -\sin\theta\cos\theta + \cos\theta\sin\theta & \sin^2\theta + \cos^2\theta\end{pmatrix}$ ***M1A1***

$= \begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}$ ***AG***

[2 marks]

Examiners report

a.i. [N/A]
a.ii. [N/A]
b. [N/A]
c. [N/A]

The transformations T_1, T_2, T_3, T_4, in the plane are defined as follows:

T_1 : A rotation of 360° about the origin
T_2 : An anticlockwise rotation of 270° about the origin
T_3 : A rotation of 180° about the origin
T_4 : An anticlockwise rotation of 90° about the origin.

The transformation T_5 is defined as a reflection in the x-axis.

The transformation T is defined as the composition of T_3 followed by T_5 followed by T_4.

a. Copy and complete the following Cayley table for the transformations of T_1, T_2, T_3, T_4, under the operation of composition of transformations. [2]

	T_1	T_2	T_3	T_4
T_1	T_1	T_2	T_3	T_4
T_2	T_2			
T_3	T_3			
T_4	T_4			

b.i. *Show that* T_1, T_2, T_3, T_4 under the operation of composition of transformations form a group. Associativity may be assumed. [3]

b.ii. Show that this group is cyclic. [1]

c. Write down the 2×2 matrices representing T_3, T_4 and T_5. [3]

d.i. Find the 2×2 matrix representing T. [2]

d.ii. Give a geometric description of the transformation T. [1]

Markscheme

a.

	T_1	T_2	T_3	T_4
T_1	T_1	T_2	T_3	T_4
T_2	T_2	T_3	T_4	T_1
T_3	T_3	T_4	T_1	T_2
T_4	T_4	T_1	T_2	T_3

A2

[2 marks]

Note: Award ***A1*** for 6, 7 or 8 correct.

b.i. the table is closed – no new elements ***A1***

T_1 is the identity ***A1***

T_3 (and T_1) are self-inverse; T_2 and T_4 are an inverse pair. Hence every element has an inverse ***A1***

hence it is a group ***AG***

[3 marks]

b.ii. all elements in the group can be generated by T_2 (or T_4) ***R1***

hence the group is cyclic ***AG***

[1 mark]

c.

T_3 is represented by $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$ ***A1***

T_4 is represented by $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ ***A1***

T_5 is represented by $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ ***A1***

[3 marks]

d.i. $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$ ***(M1)A1***

Note: Award ***M1A0*** for multiplying the matrices in the wrong order.

[2 marks]

d.ii. a reflection in the line $y = -x$ ***A1***

[1 mark]

Examiners report

a. [N/A]
b.i. [N/A]
b.ii. [N/A]
c. [N/A]
d.i. [N/A]
d.ii. [N/A]